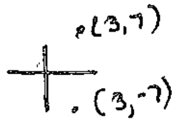
Math 1470

Name Todd & Jeana

Review for Test 1

Put answers in blank provided and **show all work**. Each problem is worth ten points.

1) If (3, 7) is on a relation that has symmetry with respect to the x-axis then give another point you know **must** be on the relation.



(3,-7)

2) Determine whether the lines are parallel, perpendicular, or neither.

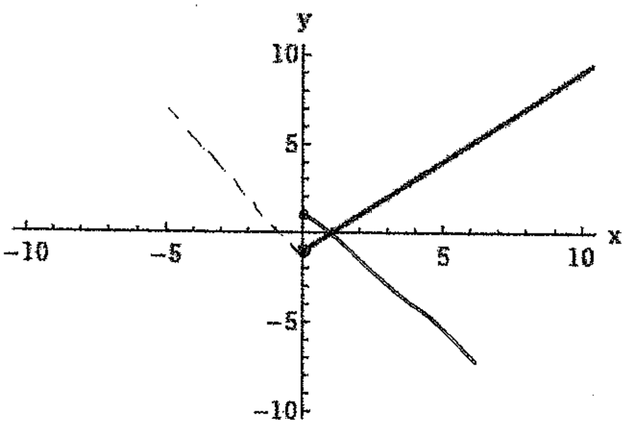
L_1: y = 5x - 4

L_2: y = 5x + 6

$m_1 = m_2$

parallel

3) Sketch the completed graph on the axes below to make the relation have symmetry with the x-axis.



Using a dashed line, sketch the completed graph on the axes above to make the function have symmetry with the y-axis.

4) Plot the points and find the distance between the two points: (0, −6) and (−4, 0).

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{(-4-0)^2+(0--6)^2}$

$\sqrt{-4^2+6^2}$

$\sqrt{14+36}$

$\sqrt{52}$

$\boxed{2\sqrt{13}}$

Find the midpoint between (0, −6) and (−4, 0).

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \left(\frac{0+-4}{2}, \frac{-6+0}{2}\right)$

$(-2,-3)$

5) Use the algebraic tests to check for symmetry with respect to both axes and the origin: $x-y^2=3$

x-axis symmetry:

y & -y on the graph

$x-(-y)^2=3$ Same as original.

$x-y^2=3$ ✓ Has x-axis symmetry

y-axis symmetry:

x & -x solve equation

$-x-y^2=3$ NOT the same as original.

Does NOT have y-axis symmetry.

origin symmetry:

BOTH

$-x-(-y)^2=3$ NOT the same as original.

$-x-y^2=3$ Does NOT have symmetry with respect to the origin.

6) Write the equation of a linear function with the values:

$f(2) = 8$ $f(7) = -1$

$\frac{Rise}{Run} = \frac{8 - -1}{2 - 7}$

$= \frac{9}{-5}$

$y = \frac{-9}{5}x + 6$

$\hookrightarrow -1 = \frac{-9}{5}(7) + 6$

$\frac{-5}{5} = \frac{-63}{5} + 6$

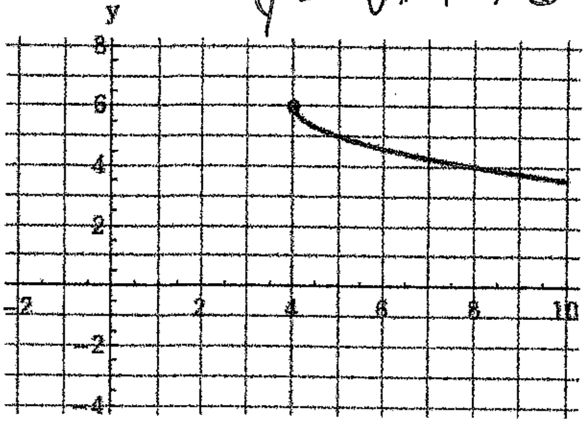
$\frac{+63}{5} = \frac{+63}{5}$

$\frac{58}{5} = 6$

$y = \frac{-9}{5} + \frac{58}{5}$

7) Using what you know about shifts and transformations, give the equation of the graph pictured

$y = -\sqrt{x-4} + 6$



8) ~~A mechanic is paid~~ \$16.00 per hour for regular time and time-and-a-half for overtime. The weekly wage function is given by the equation shown below, where *h* is the number of hours worked in a week.

$$W(h) = \begin{cases} 16h, & 0 < h \le 40 \\ 24(h-40) + 640, & h > 40 \end{cases}$$

On the back, construct a wage function for the nurse whose regular wage is \$12 per hour for the first sixty hours and time-and-a-half for overtime.

$$W(h) = \begin{cases} 12h & 0 < h \le 60 \\ 18h(h-60) + 720 & h > 60 \end{cases}$$

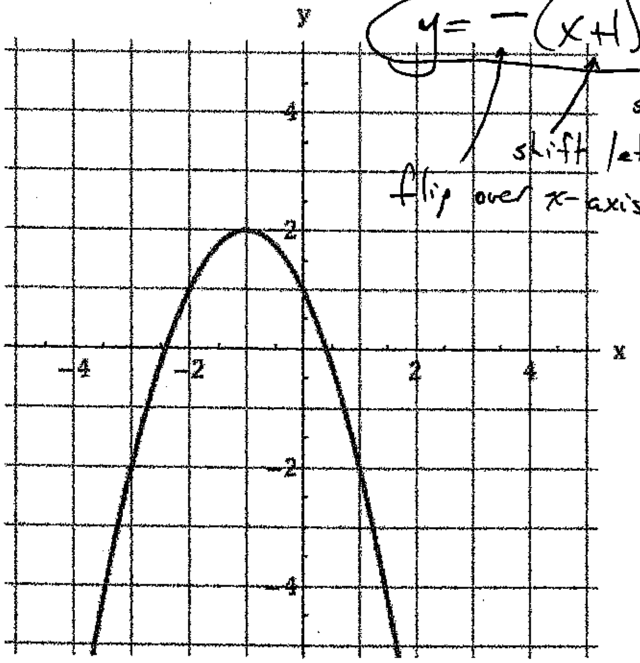
9) Use the graph of f(x) below to sketch the graph of f(-x) on the same axis.

$y = -(x+1)^2 + 2$

shift up 2

shift left 1

flip over x-axis



10) Let $f(x) = \frac{x}{x+1}$ and $g(x) = x^3$.

Find the following:

$(f+g)(x) = \frac{x}{x+1} + x^3$

$(f-g)(x) = \frac{x}{x+1} - x^3$

$(fg)(x) = \left(\frac{x}{x+1}\right)(x^3) = \frac{x^4}{x+1}$

$(f/g)(x) = \frac{x}{x+1} \div x^3 = \frac{x}{x^4+x^3}$ $\frac{1}{x^3+x^2}$

11) Let $h(x) = \sqrt[3]{x-5}$ and $j(x) = x^3 + 1$.

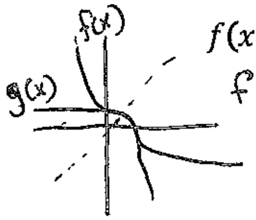
Find the following:

(hoj)(x)= $\sqrt[3]{(x^3+1)-5} = \sqrt[3]{(x^3-4)}$

(joh)(x)= $(\sqrt[3]{x-5})^3+1 = x-5+1$

$\boxed{x-4}$

12) Show that *f* and *g* are inverse functions algebraically **and** graphically (use back if necessary).



$f(x) = 1 - x^3, \quad g(x) = \sqrt[3]{1-x}$

$f(g(x)) = 1 - (\sqrt[3]{1-x})^3 = 1-(1-x) = 1-1+x = x$

13) Using algebraic long division, divide: x^3-x^2+3x+2

$(3x+2+x^3-x^2)\div(x-1)=$

$x^2 + 3 + \frac{5}{x-1}$

$x-1 \overline{)\, x^3-x^2+3x+2}$

$-(x^3-x^2)$

$0 \quad 3x+2$

$-(3x-3)$

$0 \quad 5$

$\boxed{x^2+3+\frac{5}{x-1}}$

14) Using synthetic division, divide:

$(3x+2+x^3-x^2)\div(x-1)=$

$1 \;|\; 1 \quad -1 \quad 3 \quad 2$

$\quad\;\; 1 \quad 0 \quad 3$

$1 \quad 0 \quad 3 \quad 5$

$\boxed{x^2+3+\frac{5}{x-1}}$

15) Find all roots of:

$x^4+2x^3-10x^2-18x+9=0$

possible rational $(-1,-3,-9,1,3,9)$

$-3 \;|\; 1 \quad 2 \quad -10 \quad -18 \quad 9$

$\quad -3 \quad 3 \quad 21 \quad -9$

$3 \;|\; 1 \quad -1 \quad -7 \quad 3 \quad 0$

$\quad 3 \quad 6 \quad -3$

$1 \quad 2 \quad -1 \quad 0$

x^2+2x-1

a=1

b=2

c=-1

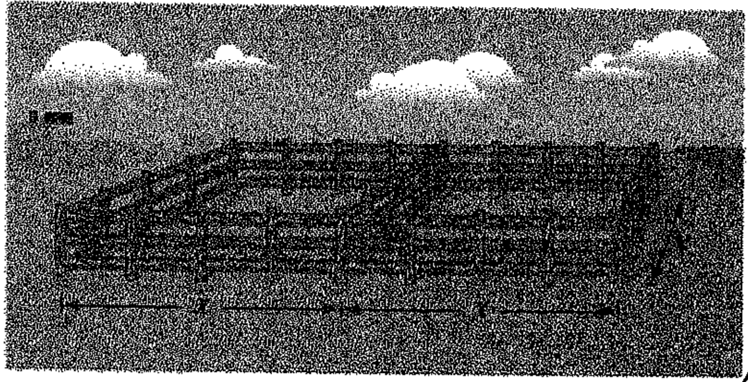
$\frac{-2\pm\sqrt{2^2-4(1)(-1)}}{2}$

$\frac{-2\pm\sqrt{4+4}}{2} = \frac{-2\pm\sqrt{8}}{2}$

$\frac{-2\pm2\sqrt{2}}{2} = -1\pm\sqrt{2}$

$\boxed{-3, 3, -1-\sqrt{2}, -1+\sqrt{2}}$

16) A rancher has 800 feet of fencing to enclose two adjacent rectangular corrals (see figure).



$800 = 3y+4x$

$y = \frac{800-4x}{3}$

Write the area *A* of the corrals as a function of *x*.

$A = 2x\left(\frac{800-4x}{3}\right)$

$\frac{2}{3}x(800-4x)$

$\frac{8}{3}x(200-x)$

$-\frac{8}{3}x(x-200)$

$-\frac{8}{3}(x^2-200x)$

$-\frac{8}{3}(x^2-200x+100^2-100^2)$

$-\frac{8}{3}[(x-100)^2-100^2]$

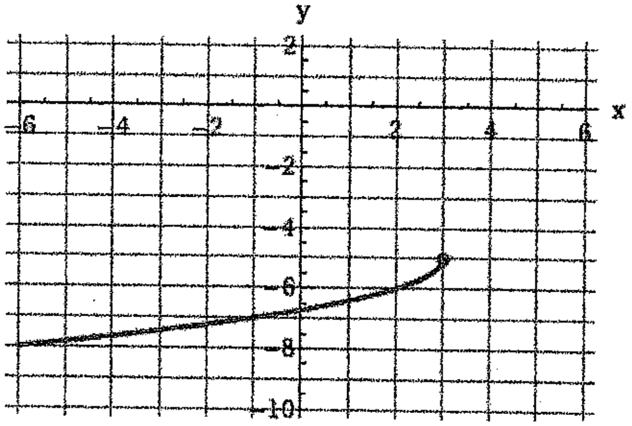
$-\frac{8}{3}(x-100)^2+\frac{80,000}{3}$

Write the area function in **standard form** to find analytically the dimensions that will produce the maximum area. (Use *A* for *f*(*x*).)

dimensions Maximumarea

$\boxed{x = 100}$

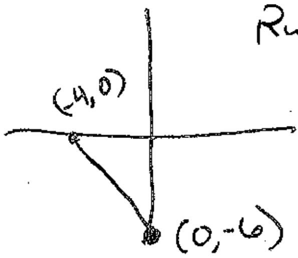
17) Using what you know about shifts and transformations, give the equation of the graph pictured



$f(x) =$ $-\sqrt{(-x+3)}\ -5$

18) Plot the points and find the slope of the line passing through the pair of points: (graph the points and the line) (0, −6), (−4, 0). Then, find the equation of the line.

Graph:



$\frac{Rise}{Run} = \frac{6 - -6}{-4-0} = \frac{6}{-4}$

$= -3/2$

$y = -3/2\,x + 6$

$-6 = -3/2(0) + 6$

$-6 = 6$

Slope: $-3/2$

Equation: $y = -3/2\,x - 6$

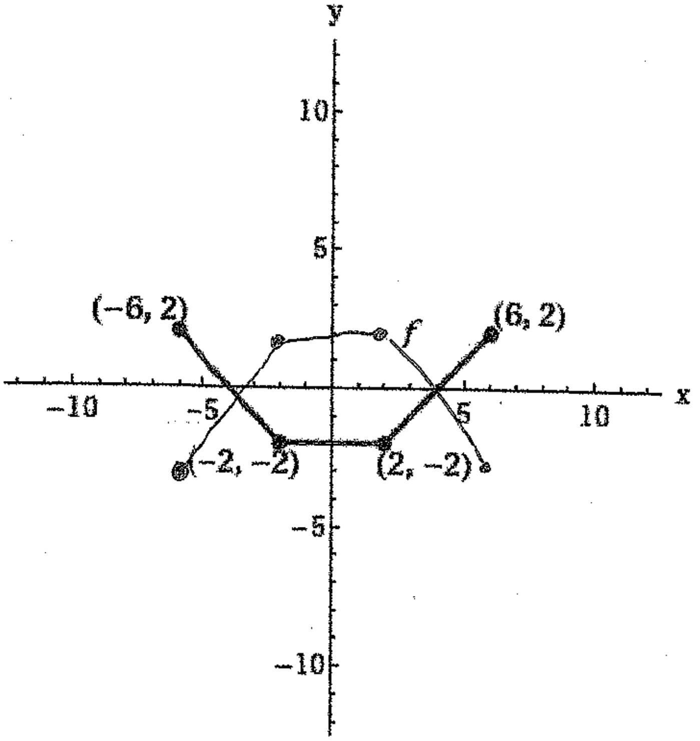
19) A mechanic is paid $16.00 per hour for regular time and time-and-a-half for overtime. The weekly wage function is given by the equation shown below, where *h* is the number of hours worked in a week.

$$W(h) = \begin{cases} 16h, & 0 < h \leq 40 \\ 24(h-40)+640, & h > 40 \end{cases}$$

Construct a wage function for the mathematician whose regular wage is $50 per hour for the first forty hours and time-and-a-half for overtime.

$$w(h): \begin{cases} 50h & 0 < h \leq 40 \\ 75(h-40)+2000 & h > 40 \end{cases}$$

20) Use the graph of -f(-x) below to sketch the graph of f(x) on the same axis.



21) Let $h(x)=\dfrac{2}{x-1}$ and $j(x)=2x+1$.

Find the following:

$(h\circ j)(x)=$ $\dfrac{2}{(2x+1)-1}$ $\quad \dfrac{2}{2x} = \dfrac{1}{x}$

$(j\circ h)(x)=$ $2\left(\frac{2}{x-1}\right)+1 = \dfrac{4}{x-1}+1$

22) Show that *f* and *g* are inverse functions algebraically **and** graphically.

$f(x)=-\frac{5}{2}x-9,\ g(x)=-\frac{2x+18}{5}$

$$f(g(x)) = f\left(\frac{-(2x+18)}{5}\right)$$
$$= \frac{-5}{2}\left(-\left(\frac{2x+18}{5}\right)\right) - 9$$
$$= \frac{2x+18}{2} - 9$$
$$= x+9-9$$
$$= x$$ ▨

$$g(f(x)) = g\left(-\tfrac{5}{2}x-9\right)$$
$$= -\frac{\left(2\left(-\frac{5}{2}x-9\right)+18\right)}{5}$$
$$= \frac{+5x+18-18}{5}$$
$$= \frac{5x}{5}$$
$$= x$$ ▨

23) Divide $(2x^5-5x^2+3)\div(x-1)$ using synthetic OR algebraic long division.

$$\begin{array}{r|rrrrrr} 1 & 2 & 0 & 0 & -5 & 0 & 3 \\ & & 2 & 2 & 2 & -3 & -3 \\ \hline & 2 & 2 & 2 & -3 & -3 & 0 \end{array}$$

$2x^4+2x^3+2x^2-3x-3$

24) Find all the **possible** rational zeros, then find all the zeros of the function: $f(x)=x^3-7x-6$

Possible zeros: { $-6, -3, -2, 1, 1, 2, 3, 6$ }

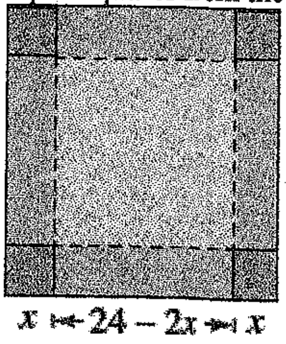
$$\begin{array}{r|rrrr} -2 & 1 & 0 & -7 & -6 \\ & & -2 & 4 & 4 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

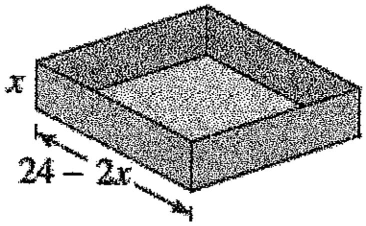
$$\begin{array}{r|rrr} -1 & 1 & -2 & -3 \\ & & -1 & 3 \\ \hline & 1 & -3 & 0 \end{array}$$

Rational zeros: { $-2,\ -1,\ 3$ }

$(x+2)(x+1)(x-3) = f(x)$

25) An open box of maximum volume is made from a square piece of material 24 cm on a side by cutting equal squares from the corners and turning up the sides.





Write a function representing the volume of the box as a function of x.

$V = x(24-2x)(24-2x)$
$= x(24-2x)^2$
$= 4x^3 - 96x^2 + 574$

Domain: $(0,12)$

Range: $(0, 1024)$